Abstract—This paper defines a new approach for cosmological simulation based on complex systems theory: a hierarchical multi-agent system is used to study stellar dynamics. At each level of the model, global behavior emerges from agent interactions. The presented model uses physically-based laws and agent-interactions to present stellar structures as the result of self-organisation. Nevertheless, a strong bond with cosmology is kept by showing the capacity of the model to exhibit structures close to those of the observable universe.

I. INTRODUCTION

For years, numerical simulation in cosmology has tried to reproduce and explain the wide variety of patterns (from globular clusters to spiral galaxies) and complex behaviors that cosmological evolution shows. Algorithms used are based on strictly reductionnist models (such as [1] [2]).

Even if this approach has carried out to successes, some problems remain unsolved:

- Observed dynamics highly depend on the number of point-mass particles used in simulation [3][4](strict application of physical laws requires the use of point-mass particles in simulation models). Using this approach, some complex patterns do not appear in the cosmological models till a high number of particles (over $10^{24}$).

- In addition, a realistic number of point-mass particles should be around $10^{41}$ [5] for a typical spiral galaxy what is a calculative impossibility.

The current approach to solve these problems is to define increasingly precise models taking in account more and more parameters [6]: each physical phenomenon is calculated by a dedicated algorithm (such as gravity [7] [8]) and the results are combined according to the goal of the experiment. The average number of particles currently used is between $128^3$ and $512^3$.

In the same time, the study of complex systems obtained a certain success, in particular by the use of models like cellular automata [9] [10] [11]. Even if results get by these models are structurally close to the observable, they are often too abstract and too far away from physical reality to be easily accepted by the cosmologists community.

Based on previous works [12], our approach is to set up of a multi-agent model using a multi-layers system to mitigate the calculative problem. The model is using physical laws as intra-levels interactions and agents at a given level are aggregated to form structures of higher level, subjected to another physical law.

This aim of this paper is to present and investigate a new kind of model to show that such models needs a lower number of agents than what is necessary for cosmological models (from a calculative point of view) to exhibit observable structures.

II. PREAMBLE

Gravitation is the main law of all celestial mechanics in the majority of the theories and thus takes a significant part in the models which intend to simulate this dynamics:

$$\vec{F}_{ij} = -G \cdot \frac{m_i m_j}{r_{ij}^2}$$  \hspace{1cm} (1)

with:

- $G$ the gravitationnal constant.
- $m_k$ the mass of the agent $k$.
- $r_{ij}$ the distance between agent $i$ and agent $j$.

Few algorithms exist to calculate the impact of the gravitation force (1) on the various elements of a system. Presented by Barnes & Hut [7] the TreeCode is one of these algorithms designed to calculate the effect of gravity. It was proposed to obtain a significant number of particles and a correct resolution.

This algorithm allows to specify a parameter $\theta$ which determines from which distance the action of $n$ close particles is not dissociable from the action of a particle having the added mass of these $n$ particles. Its use in a wide number of cosmological algorithm and its adaptability justifies its use in our model.

III. HIERARCHICAL MULTI-AGENT MODEL

The obvious complexity of the problem and wide variety of forces and scales [6] led us to use of a hierarchical multi-agent system. The aim is to define a high-level model, highly parametrizable to get a picture of universe and to study how
cosmological patterns can emerge from interaction within and between complexity levels.

A. Overview

Our model is composed by four kinds of agents and complexity levels:

- **Level 1 agent**: dedicated to internal interactions, this is an abstraction of mass matter subjected to gravitational attraction.

- **Level 2 agent**: dedicated to local interactions and interactions applying on a short distance (approximated as a vicinity) such as accretion.

- **Level 3 agent**: dedicated to long range interactions and forces which apply without any distance constraints such as gravity.

- **Level 4 agent**: dedicated to environmental actions: all the forces which apply to the whole agents in the universe, such as expansion for instance.

Schematic representation of our hierarchical multi-agent model:

The structure formed at the end of all these experiments is spherical and corresponds to what we defined as the vicinity of level one agents. To calculate the evolution of the radius, a gravity force will be applied to point-mass particles by using this equation:

$$F = G \cdot \frac{m}{r^2}$$

with:

- $m$ the global mass of the agent
- $r$ the radius of the agent.

The algorithm, widely used in cosmology, presented above will be used as a basis for the gravity application. By correlating the experiments we can realize that it exists in all simulations a transitory series of state (limited in the time or not) during which the particles collapse on themselves. The duration and the collapsing speed (from stabilization to total collapsing) are function of the relationship between the overall mass of the system and the number of agents on which it is distributed (Fig.1). These results correspond to the evolution function of the radius applied on peripheral elements: agents are approximated as a sphere whose radius and spin varies according to the inner. They can represent a mass of gas, purely gravitational matter (called dark matter) or a star.

At this level the evolving parameters are:

- **mass** $m_1$
- **position** $(x_1, y_1, z_1)$
- **radius** $r_1$

Radius represents the size of the spherical structure.

**Agents exchange**: Each agent communicates, to all the other agents in its vicinity, its position in space and its mass to allow the calculation of the generated gravitational force.

**Level 2: Agents description**

Level two gather all the interactions applying on a short distance such as accretion [13]. Therefore the function 3 is applied by this level of complexity to get the velocity.
modification $\delta V_x^i$:

$$\delta V_x^i = \sum_{j=1}^{M} \left[ \mu_j^i \cdot (V_j^i(x) + s_j^i(x)) \right]$$  \hspace{1cm} (3)

$s_j^i$ : the spin of the agent $j$ at time $t$

with:

$V_j^i(x)$ : velocity of the agent $j$ at time $t$

$\mu_j^i$ : the accretion capacity of the agent $j$ at time $t$

Fig.2 shows examples of this concept of vicinity and its physical mapping: structures, self-formed by the effect of gravity, are surrounded of a less dense matter halo on which local forces, such as accretion power, apply.

At this level the evolving parameters are:

- mass $m_2$
- position $(x_2, y_2, z_2)$
- spin $(S_x, S_y, S_z)$
- velocity $(V_{x2}, V_{y2}, V_{z2})$
- radius $r_2$

Radius represents the size of the vicinity.
Velocity represents the linear velocity.
Spin represents the revolution speed.

Agents exchange:

Each agent communicates, to all the other agents in its vicinity, its linear velocity and its number of revolutions (which we called spin) in order to calculate the impact of accretion forces.

Level 3: Agents description:

Regarded as the principal force responsible for cosmological structures formation, gravity applies to the whole mass elements present in the universe. It is described by the equation:

$$\frac{dx_i}{dt^2} = G \cdot \sum_{i,j=1}^{M} \left( \frac{m_j (x_j - x_i)}{(d_{ij})^3} \right)$$  \hspace{1cm} (4)

$G$ : gravitational constant
$m_i$ : mass of the agent $i$
$d_{ij}$ : range between agent $i$ and agent $j$

Level 4: Agents description:

The universe, as we know it, is expanding. This can be simulated as a force applied to all the agents and aiming at moving away all elements of the system. The expansion will be set as a radial force applied by the environment to all agents, approximating the formula:

$$v = H \cdot r$$  \hspace{1cm} (5)

$H$: Hubble’s constant
$v$: resulting velocity of an unspecified point of the universe
$r$: radius of curvature of the universe

Agents exchange:

The agents do not communicate, they are simply subjected to the environmental forces.

C. Inter-levels exchange

Each level $N$ agent transmits, to all level $N-1$ to level $N-3$ agents (if they exist), the whole variations it was subjected to. For instance, if the position $(X_3, Y_3, Z_3)$ of a level 3 agent is modified $(\delta x, \delta y, \delta z)$, the position $(X_2, Y_2, Z_2)$ of the whole level 2 agents, contents in this agent, evolves by $(\delta x, \delta y, \delta z)$, just as the position $(X_1, Y_1, Z_1)$ of level 1 agents.

D. Static modifiers

In addition to the elements above, the model contains static parameters, applied to the results of the equations above:

Level 1:
- gravity modifier $g_1$

Level 2:
- global accretion power $\mu$
Level 3:
- gravity modifier $g_3$
Level 4:
- expansion power $e$

These modifiers are used to widen and facilitate the system parameterization. They are hand-fixed at the beginning of an experiment and do not evolve till the end.

Gravity modifier ($g_1$ and $g_3$) are used to take into account some physical theories using different kind of gravitation according to the studied celestial object.

Accretion power parameter allows to modify, even remove, the viscosity of the whole universe.

Expansion power vary the force of expansion to take into account various assumptions: an expanding universe accelerated, a fixed universe or a universe breaking down on itself.

IV. EXPERIMENTAL VALIDATION

A. Model Validation

In order to validate the model defined above, we show that the emergent structures are coherent with the observable ones. In this experiment we simulate the collision of two particular galaxies (identified as G1 and G2) and compare it with other simulation and real pictures from the spatial telescope Hubble. The structure, called “The Mice”, is often studied in cosmology [14] [15] [16]. The experiment is undertaken with the following parameters [14]:

\[
\begin{align*}
\text{mass (G1)} &= 3.95 \times 10^{41} \text{ kg} / \text{mass (G2)} = 4.05 \times 10^{41} \text{ kg} \\
\text{radius (G1)} &= 9.40 \text{ kpc} / \text{radius (G2)} = 11.0 \text{ kpc} \\
1 \text{s “simulation time”} &= 1.10^{14} \text{s “cosmological time”}
\end{align*}
\]

The observational data indicate that the collision took place 160 Myr\(^2\) ago. To get the image of simulation corresponding to the current state of these galaxies, it is necessary to let evolve simulation during 220 Myr (which is equivalent to 69s in simulation time with our system). The distribution of the elements is as follows [14]: 50% of gas (represented in yellow), viscous and subjected to gravitation forces, and 50% of dark matter (represented in white), only affected by gravity. Fig.3 shows the result of this evolution and a comparison with other data.

![Fig. 3. 1) shows the evolution of our model with 300,000 agents. 2) shows the result of a simulation using dedicated cosmological algorithm by J. Hibbard. 3) shows a picture get by the spatial telescope Hubble (http://hubblesite.org/).](image)

These results are obtained after the same evolution time and show that the dynamics and the resulting emergent patterns are the same.

B. Experimental Protocol

To check if our model is less dependent of the number of elements used than a classical model, a series of experiments aiming at repeating the same simulation with a decreasing number of agents/particles has been carried out.

Six simulations have been led using: 100, 1,500, 3,000, 10,000, 30,000, 100,000 and 300,000 agents with our hierarchical model and a classical Treecode. Same experimental conditions are used in each simulation (as describe in IV-A).

All the static modifier are set to 1 (neutral value) in these experiments.

C. Results

Fig.4 presents the same kinds of experiments conducted with a small number of agents and the various generated dynamic.

![Fig. 4. Left, the result of a simulation using dedicated cosmological algorithm (TreeCode) with 3,000 particles. Right, the same simulation time with our model (still 3,000 agents). Dark matter in red. Gas in blue.](image)

With such few agents (compare to classical cosmological simulations), a structure, qualitatively similar to “The Mice”,

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\(^1\)1kpc = 3.08568025 \times 10^{19} \text{ m} \\
\(^2\)1 Myr = 1 \text{ million years} = 3.1536 \times 10^{13} \text{s}
emerge from our model. Same experiment with a TreeCode does not lead to any stable structure and all the agents are scattered in the universe.

Fig. 5 show the difference of behavior according to the model used: the left row shows the final step of evolution with various densities of agents (1 500, 3 000 and 100 000) and the right row shows same experiments using a TreeCode.

![Simulation Images]

The experiment with 1 500 agents does not lead to the formation of any stable structure: dynamics is linear and the agents scatter progressively. Dark matter and gas form aggregates of matter, visible on the picture. Experiments with 3 000 and 100 000 show the same dynamic: after G1 and G2 collision both structures remain compact and it appears a "tail" of agents because of their inertia, but they are still subjected to the gravitation of the central mass. In these experiments, dark matter remains distinct from gas and forms a halo around the central structure. The size of this halo is close to the gas structure one, even if, in the experiment with 100 000, as we can see, many dark matter particles escape from it.

In the experiment with 1 500 and 3 000 agents we find the same spatial distribution of the agents as with the hierarchical model in the experiment with 1 500 agents: gas and dark matter aggregate and scattered in all the universe. It is only with 100 000 agents that dynamics, carrying out to the formation of a structure similar to “The Mice”, is found.

V. DISCUSSION

Results presented above qualitatively show that, in contrast with classical cosmological models, beyond a threshold, the number of elements does not influence dynamics any more. Such a threshold seems to be around 2 000 elements in this experiments but further studies need to be done to specify the conditions and the reasons of appearance of this threshold value.

In contrast to the physical models where a reduction of the number of particles leads to a less homogeneous application of forces, the evolution of the agents at each level on our model compensate this drift: in both models point-mass particles (level 1 agent in our model) are be subjected to less important gravity forces. In a traditional model that leads to an expansion dynamics of all the elements. In the hierarchical model this leads to an increase of the size of the level 2 agents and the size of the vicinity of these agents. Such a increase modify the importance and the range of the transition functions applied by this level (accretion). These transition functions increase the total cohesion between agents, compensating grativity and improving formation and survival of complex structures.

VI. CONCLUSION

In this paper a new hierarchical multi-agent model aiming to solve the inherent problems of the point-mass particle approach used by the cosmologists have been introduced. Qualitative evidence have been presented that a hierarchical multi-agent model is less sensitive to the number of elements used for a simulation than traditional models: beyond some threshold, emergent dynamics remain the same. Future works include a quantitative analysis of the results presented here as well as a study of the complexity classes described by a wide parametrization of this model: by replacing the universe as-we-know-it in a larger picture of universe as-it-could-be, we will try to understand patterns formation and dynamical evolution. As it exists a difference between observation and numerical simulation on spheroidal galactic formation (radial velocity of peripheral elements does not match with the observable one [17]) we will check if the model defined in this paper can bring some answers.

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