AN EMPIRICAL CENTRAL LIMIT THEOREM FOR INTERMITTENT MAPS

J. DEDECKER[1]

Abstract. We prove an empirical central limit theorem for the distribution function of a stationary sequence, under a dependence condition involving only indicators of half line. We show that the result applies to the empirical distribution function of iterates of expanding maps with a neutral fixed point at zero as soon as the correlations are summable.

Classification MSC 2000. 37E05, 60F17, 60G10.

Short Tilte: Empirical CLT for intermittent maps

Key words. Intermittency, empirical distribution function, Rosenthal inequalities.

1. Introduction

For γ in $]0,1[$, we consider the intermittent map T_γ from $[0,1]$ to $[0,1]$, introduced by Liverani, Saussol and Vaienti (1999):

$$T_\gamma(x) = \begin{cases} x(1+2^\gamma x^\gamma) & \text{if } x \in [0,1/2[\\ 2x-1 & \text{if } x \in [1/2,1] \end{cases}$$

We denote by ν_γ the unique T_γ-invariant probability measure on $[0,1]$ which is absolutely continuous with respect to the Lebesgue measure.

In 1999, Young showed that such systems (among many others) may be described by a Young tower with polynomial decay of the return time. From this construction, she was able to control the covariances $\nu_\gamma(g \circ T^n \cdot (f - \nu_\gamma(f)))$ for any bounded function g and any α-Hölder function f, and then to prove that

$$\frac{S_n(f)}{\sqrt{n}} = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} (f \circ T_\gamma^i - \nu_\gamma(f))$$

converges in distribution to a normal law as soon as $\gamma < 1/2$. When $\gamma = 1/2$, Gouëzel (2004) proved that if f is α-Hölder, then $(n\ln(n))^{-1/2} S_n(f)$ converges to a normal distribution with mean 0 and variance $h(1/2)(f(0) - \nu_{1/2}(f))^2$, where h is the density of $\nu_{1/2}$.

Let K_γ the Perron-Frobenius operator of T_γ with respect to ν_γ: for any bounded measurable functions f, g,

$$\nu_\gamma(f \cdot g \circ T_\gamma) = \nu_\gamma(K_\gamma(f) g)\,. \tag{1.1}$$

Let $(X_i)_{i\geq 0}$ be a stationary Markov chain with invariant measure ν_γ and transition Kernel K_γ. It is well known (see for instance Lemma XI.3 in Hennion and Hervé (2001)) that on the probability space $([0,1], \nu_\gamma)$, the random variable $(T_\gamma, T_\gamma^2, \ldots, T_\gamma^n)$ is distributed as $(X_n, X_{n-1}, \ldots, X_1)$. Hence any information on the law of $S_n(f)$ can be obtained by studying the law of $\sum_{i=1}^n (f(X_i) - \nu_\gamma(f))$.

[1] Université Paris 6-Pierre et Marie Curie, Laboratoire de Statistique Théorique et Appliquée.

In a recent paper, Dedecker and Prieur (2008) have computed some dependence coefficients of the Markov chain $(X_i)_{i\geq 0}$. As a consequence they obtain that, if $\gamma < 1/2$ and f is any bounded variation (BV) function, then $n^{-1/2}(S_n(f)-\nu_\gamma(f))$ converges in distribution to a normal law. A natural question is then: for $\gamma < 1/2$, can we prove a uniform central limit theorem over the class of BV functions whose variation is less than 1? Or equivalently, can we prove that the empirical process

$$\Big\{\frac{1}{\sqrt{n}}\sum_{k=1}^{n}\big(\mathbf{1}_{T_\gamma^k\leq t}-\nu_\gamma([0,t])\big), t\in[0,1]\Big\}$$

converges in the space $\ell^\infty([0,1])$ of bounded functions from $[0,1]$ to $\mathbb{R}$ (equipped with the uniform norm) to a Gaussian process?

In this paper, we give a positive answer to this question: the empirical central limit theorem holds as soon as $\gamma < 1/2$. Let us briefly recall the previous results for uniformly expanding maps T. Assume that there is a finite partition $\{I_1,\ldots,I_N\}$ of $[0,1]$ into intervals of continuity and monotonicity of T. Assume moreover that the absolutely continuous T-invariant probability measure μ is unique and that (T,μ) is weakly mixing. If $|T'|\geq\lambda>1$ on any interval of the partition, the empirical central limit theorem follows from Theorem 5 applied to Example 1.4 in Borovkova, Burton and Dehling (2001). Under the weaker assumption that there exist $A>0$ and $\lambda>1$ such that $|(T^n)'|\geq A\lambda^n$ for any positive integer n on any interval of the partition associated to T^n, the empirical central limit theorem is due to Collet, Martinez and Schmitt (2004). In Dedecker and Prieur (2007), Section 6.3, the assumption on the finite partition has been removed. Note that our main result (Theorem 2.1 of Section 2) also applies in that case.

At this point, a question remains open: can we get an empirical central limit theorem for $\gamma = 1/2$ under the normalization $(n\ln(n))^{1/2}$? Starting from Gouëzel's result (2004) for $\gamma = 1/2$, one can prove (see the appendix) that the finite dimensional marginals of the empirical process (with normalization $(n\ln(n))^{1/2}$) converge to those of a degenerated Gaussian process G defined by:

$$\text{for any } t\in[0,1],\quad G(t)=\sqrt{h(1/2)}(1-F_{\nu_{1/2}}(t))\mathbf{1}_{t\neq 0}Z\,,$$

where $F_{\nu_{1/2}}$ is the distribution function of $\nu_{1/2}$, and Z is a standard Gaussian. A reasonable conjecture is that the convergence holds in $\ell^\infty([0,1])$. If this is true, the tightness of the empirical process must hold with respect to the natural metric $\rho(s,t)=\|G(t)-G(s)\|_2$, or equivalently to the metric $d(s,t)=|s-t|$ for s,t in $]0,1]$, and $d(0,t)=|1-t|$ for t in $]0,1]$.

The paper is organized as follows. In Section 2, we prove an empirical central limit theorem (Theorem 2.1) for a strictly stationary sequence of real valued random variables, under the condition that the coefficient $\beta_2(k)$ defined in Definition 2.1 is such that $\beta_2(k)=O(k^{-1-\delta})$ for some $\delta>0$. As a consequence, we give in Corollary 2.1 the empirical central limit theorem for the iterates of T_γ, when $\gamma<1/2$. The proof of Theorem 2.1 is based on a a new Rosenthal inequality for sums of random variables having moments of order $p\in[2,3]$, which is stated and proved in Section 3. In Proposition 4.1 of the appendix, we prove the finite dimensional convergence of the empirical process in the case $\gamma=1/2$.

2. An empirical central limit theorem for stationary sequences

Let $(X_i)_{i\in\mathbb{Z}}$ be a stationary sequence of real valued random variables, with common distribution function F, and let

$$F_n(t) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}_{X_i\le t} \,.$$

Let $\mathcal{M}_l = \sigma(X_i, i\le l)$ and $\mathcal{M}_{-\infty} = \cap_{i\in\mathbb{Z}}\mathcal{M}_i$. Applying the central limit theorem given in Gordin (1973), it is easy to see that the finite dimensional marginals of $\sqrt{n}(F_n - F)$ converge to those of a Gaussian process G as soon as,

(1) for any $s,t\in\mathbb{R}$, $\quad \mathbb{E}(\mathbf{1}_{X_i\le t}\mathbf{1}_{X_j\le s}|\mathcal{M}_{-\infty}) = \mathbb{E}(\mathbf{1}_{X_i\le t}\mathbf{1}_{X_j\le s})$.

(2) for any $t\in\mathbb{R}$, $\quad \sum_{k>0}\|\mathbb{E}(\mathbf{1}_{X_k\le t}|\mathcal{M}_0) - F(t)\|_1 < \infty$.

In view of (2), if one wants to prove the convergence in distribution of $\sup_{t\in\mathbb{R}}\sqrt{n}|F_n(t)-F(t)|$ to $\sup_{t\in\mathbb{R}}|G(t)|$, it seems natural to impose some conditions on

$$\alpha(k) = \sup_{t\in\mathbb{R}}\|\mathbb{E}(\mathbf{1}_{X_k\le t}|\mathcal{M}_0) - F(t)\|_1 \,. \tag{2.1}$$

Note that this coefficient $\alpha(k)$ is weaker than the usual strong mixing coefficient of Rosenblatt (1956). According to Rio's result (2000, Theorem 7.2) for strongly mixing sequences, one can wonder if the empirical central limit theorem holds provided that (1) holds, and

$$\alpha(n) = O(n^{-1-\delta}) \quad \text{for some } \delta > 0.$$

We shall see in Theorem 2.1 below that this result is true provided that $\alpha(k)$ is replaced by a stronger coefficient $\beta_2(k)$, introduced in Dedecker and Prieur (2007). The difference between the definition (2.1) and that of $\beta_2(k)$ is that one can control any products $(\mathbf{1}_{X_i\le t} - F(t))(\mathbf{1}_{X_j\le s} - F(s))$, and the supremum is s,t is taken before the expectation.

Let us define this coefficient more precisely:

Definition 2.1. *Let P be the law of X_0 and $P_{(X_i,X_j)}$ be the law of (X_i,X_j). Let $P_{X_k|X_0}$ be the conditional distribution of X_k given X_0, $P_{X_k|\mathcal{M}_l}$ be the conditional distribution of X_k given $\mathcal{M}_l$, and $P_{(X_i,X_j)|\mathcal{M}_l}$ be the conditional distribution of (X_i,X_j) given $\mathcal{M}_l$. Define the functions $f_t = \mathbf{1}_{]-\infty,t]}$, and $f_t^{(0)} = f_t - P(f_t)$. Define the random variables*

$$\begin{aligned}
b(X_0,k) &= \sup_{t\in\mathbb{R}} |P_{X_k|X_0}(f_t) - P(f_t)| \,, \\
b_1(\mathcal{M}_l,k) &= \sup_{t\in\mathbb{R}} |P_{X_{k+l}|\mathcal{M}_l}(f_t) - P(f_t)| \,, \\
b_2(\mathcal{M}_l,k) &= \sup_{i>j\ge k+l}\ \sup_{(s,t)\in\mathbb{R}^2} |P_{(X_i,X_j)|\mathcal{M}_l}(f_t^{(0)}\otimes f_s^{(0)}) - P_{(X_i,X_j)}(f_t^{(0)}\otimes f_s^{(0)})| \,.
\end{aligned}$$

Define now the coefficients

$$\beta(\sigma(X_0),X_k) = \mathbb{E}(b(X_0,k)), \quad \beta_1(k) = \mathbb{E}(b_1(\mathcal{M}_0,k)) \quad \textit{and} \quad \beta_2(k) = \max\{\beta_1(k), \mathbb{E}((b_2(\mathcal{M}_0,k)))\} \,.$$

As usual, we denote by $\ell^\infty(\mathbb{R})$ the space of bounded functions from $\mathbb{R}$ to $\mathbb{R}$ equipped with the uniform norm. For details on weak convergence on the non separable space $\ell^\infty(\mathbb{R})$, we refer to van der Vaart and Wellner (1996) (in particular, we shall not discuss any measurability problems, which can be handled by using the outer probability).

Our main result is the following:

Theorem 2.1. *Assume that F is continuous. If $\beta_2(n) = O(n^{-1-\delta})$ for some $\delta > 0$, then $\sqrt{n}(F_n - F)$ converges in distribution in $\ell^\infty(\mathbb{R})$ to a centered gaussian process G, whose sample paths are almost surely uniformly continuous with respect to the pseudo-metric*

$$d(x, y) = |F(x) - F(y)| .$$

Moreover, the covariance function of G is given by

$$\mathrm{Cov}(G(s), G(t)) = \sum_{k\in\mathbb{Z}} \mathrm{Cov}(\mathbf{1}_{X_0\le t}, \mathbf{1}_{X_k\le s}) .$$

Remark 2.1. *Theorem 2.1 improves on the corresponding result in Dedecker and Prieur (2007), who assumed that $\beta_2(n) = O(n^{-2-\delta})$ for some $\delta > 0$. We refer to Section 6 in Dedecker and Prieur (2007) for many examples of stationary processes for which the coefficients $\beta_2(k)$ can be computed.*

Let us give the application of Theorem 2.1 to the iterates of T_γ. We keep the same notations as in the introduction.

Corollary 2.1. *Let $F_{\nu_\gamma}(t) = \nu_\gamma([0, t])$ and $F_{n,\gamma}(t) = n^{-1}\sum_{i=1}^n \mathbf{1}_{T_\gamma^i\le t}$. Assume that $\gamma \in]0, 1/2[$. On the probability space $([0, 1], \nu_\gamma)$ the process $\sqrt{n}(F_{n,\gamma} - F_{\nu_\gamma})$ converges in distribution in $\ell^\infty([0, 1])$ to a centered Gaussian process G_γ, whose sample paths are almost surely uniformly continuous. Moreover the covariance function of G_γ is given by*

$$\mathrm{Cov}(G_\gamma(s), G_\gamma(t)) = \nu_\gamma(f_t^{(0)} \cdot f_s^{(0)}) + \sum_{k>0} \nu_\gamma(f_t^{(0)} \cdot f_s^{(0)} \circ T_\gamma^k) + \sum_{k>0} \nu_\gamma(f_s^{(0)} \cdot f_t^{(0)} \circ T_\gamma^k) , \tag{2.2}$$

where the function $f_t^{(0)}$ is defined by $f_t^{(0)}(x) = \mathbf{1}_{x\le t} - \nu_\gamma([0, t])$.

Remark 2.2. *Let us give here another application of Theorem 2.1. Let $X_i = \sum_{i\ge 0} a_k\varepsilon_{i-k}$ where $(\varepsilon_i)_{i\in\mathbb{Z}}$ is a sequence of i.i.d. random variables such that $\mathbb{E}(|\varepsilon_0|^\alpha) < \infty$ for some $\alpha \in]0, 1[$, and $a_i = O(\rho^i)$ for some $\rho \in]0, 1[$. Let w be the modulus of continuity of F. Following the computations made in Section 6.1 in Dedecker and Prieur (2007), we obtain that*

$$\beta_2(k) \le 2w(x) + K\Big(\frac{\rho^n}{x}\Big)^\alpha ,$$

for some $K > 0$ and any $x > 0$. Taking $x = \rho^n n^{2/\alpha}$, we infer that $\beta_2(n) = O(n^{-1-\delta})$ for some $\delta > 0$ as soon as

$$w(x) \le C|\ln(x)|^{-a} \quad \text{in a neighbourhood of 0, for some } a > 1 . \tag{2.3}$$

Hence Theorem 2.1 applies as soon as the modulus of continuity of F satisfies (2.3). This result improves on Corollary 4.2 in Dehling, Durieu and Volnỳ (2007), who required $a > 2$ in (2.3), and $\|\varepsilon_0\|_\infty < \infty$.

Proof of Theorem 2.1. For any strictly stationary sequence $V = (V_i)_{i\in\mathbb{Z}}$ of real valued random variables, let

$$\mu_{n,V}(t) = \frac{1}{\sqrt{n}}\sum_{k=1}^n \big(\mathbf{1}_{V_i\le t} - \mathbb{P}(V_0 \le t)\big) .$$

Let P be the law of X_0, and let Q be the probability on $\mathbb{R}$ whose density with respect to P is

$$\frac{1 + 4\sum_{k=1}^\infty b(x, k)}{1 + 4\sum_{k=1}^\infty \beta(\sigma(X_0), X_k)} .$$

Let F_Q be the distribution function of Q, and note that F_Q is continuous because F is continuous. Let $Y_i = F_Q(X_i)$. Clearly $\mu_{n,X} = \mu_{n,Y} \circ F_Q$ almost surely. Since F_Q is non decreasing, the β-dependence coefficients of $(Y_i)_{i\in\mathbb{Z}}$ are smaller than those of $(X_i)_{i\in\mathbb{Z}}$. Arguing as in the proof of Theorem 1 in Dedecker and Prieur (2007), since $\sum_{k>0}\beta_2(k) < \infty$, we obtain that the finite-dimensional marginals of $\mu_{n,Y}$ converges to those of the Gaussian process W with covariance function

$$\mathrm{Cov}(W(s), W(t)) = \sum_{k\in\mathbb{Z}} \mathrm{Cov}(\mathbf{1}_{Y_0\le t}, \mathbf{1}_{Y_k\le s}) .$$

Let us check that one can choose W such that its sample paths are almost surely uniformly continuous with respect to the usual distance on $[0,1]$. By definition of $b(X_0,k)$ one has, for $0\le s\le t\le 1$,

$$|\mathrm{Cov}(\mathbf{1}_{s<Y_0\le t}, \mathbf{1}_{s<Y_k\le t})| = |\mathbb{E}(\mathbf{1}_{s<Y_0\le t}\mathbb{E}(\mathbf{1}_{s<Y_k\le t} - \mathbb{E}(\mathbf{1}_{s<Y_k\le t})|X_0))| \le 2\mathbb{E}(\mathbf{1}_{s<Y_0\le t} b(X_0,k)) .$$

From this upper bound and the fact that, for any Q-distributed random variable V, $F_Q(V)$ is uniformly distributed over $[0,1]$, we infer that

$$\mathrm{Var}(W(t)-W(s)) \le \Big(1+4\sum_{k>0}\beta(\sigma(X_0),X_k)\Big)\int \mathbf{1}_{s<F_Q(x)\le t} Q(dx) = \Big(1+4\sum_{k>0}\beta(\sigma(X_0),X_k))\Big)(t-s) .$$

This is enough to ensure that there exists a Gaussian process W whose sample paths are almost surely uniformly continuous with respect to the usual distance on $[0,1]$.

Consequently, the sample paths of the Gaussian process $G = W\circ F_Q$ are almost surely uniformly continuous with respect to the distance d_Q defined by $d_Q(x,y) = |F_Q(x)-F_Q(y)|$. Now, the uniform continuity with respect to d_Q is equivalent to the uniform continuity with respect to d, since P and Q are equivalent.

To prove Theorem 2.1, it suffices to prove that $\mu_{n,Y}$ converges in distribution in $\ell^\infty([0,1])$ to W. In fact, it remains to prove the tightness, that is, for any $\epsilon>0$,

$$\lim_{\delta\to 0}\limsup_{n\to\infty}\mathbb{P}\Big(\sup_{|s-t|\le\delta}|\mu_{n,Y}(t)-\mu_{n,Y}(s)|>\epsilon\Big) = 0 . \tag{2.4}$$

For $x\in[0,1]$ and any positive integer K, let $\Pi_K(x) = 2^{-K}[2^K x]$. Clearly (2.4) will follow from the tightness of W and from the convergence of the marginals of $\mu_{n,Y}$ to those of W, if we can prove that

$$\lim_{K\to\infty}\limsup_{n\to\infty}\mathbb{E}\Big(\sup_{x\in[0,1]}|\mu_{n,Y}(x)-\mu_{n,Y}(\Pi_K(x))|\Big) = 0 . \tag{2.5}$$

In the rest of the proof, we shall prove (2.5). To simplify the notations, let $\mu_n = \mu_{n,Y}$, and $Z_n = d\mu_n$. In the following, C is a positive constant which may vary form line to line.

We first use the elementary decomposition

$$\mu_n(x) - \mu_n(\Pi_K(x)) = \sum_{L=K+1}^{M} \mu_n(\Pi_L(x)) - \mu_n(\Pi_{L-1}(x)) + \mu_n(x) - \mu_n(\Pi_M(x)) .$$

Consequently,

$$\sup_{x\in[0,1]}|\mu_n(x)-\mu_n(\Pi_K(x))| \le \sum_{L=K+1}^{M}\Delta_L + \Delta_M^* , \tag{2.6}$$

where

$$\Delta_L = \sup_{1\le i\le 2^L} |Z_n(](i-1)2^{-L}, i2^{-L}])| \quad \text{and} \quad \Delta_M^* = \sup_{x\in[0,1]} |Z_n(]\Pi_M(x), x])| \,.$$

Note that

$$-\sqrt{n}\mathbb{P}(\Pi_M(x) < Y_0 \le \Pi_M(x) + 2^{-M}) \le Z_n(]\Pi_M(x), x]), \tag{2.7}$$

and

$$Z_n(]\Pi_M(x), x]) \le Z_n(]\Pi_M(x), \Pi_M(x) + 2^{-M}]) + \sqrt{n}\mathbb{P}(\Pi_M(x) < Y_0 \le \Pi_M(x) + 2^{-M}) \,. \tag{2.8}$$

Let $C(\beta) = 1 + 4\sum_{k=1}^\infty \beta(\sigma(X_0), X_k)$. Clearly

$$\mathbb{P}(\Pi_M(x) < Y_0 \le \Pi_M(x) + 2^{-M}) \le C(\beta)\int \mathbf{1}_{\Pi_M(x)<F_Q(x)\le \Pi_M(x)+2^{-M}} Q(dx) = C(\beta)2^{-M} \,. \tag{2.9}$$

From (2.7), (2.8) and (2.9), we infer that $\Delta_M^* \le \Delta_M + C(\beta)\sqrt{n}2^{-M}$. Consequently, it follows from (2.6) that

$$\sup_{x\in[0,1]} |\mu_n(x) - \mu_n(\Pi_K(x))| \le C(\beta)\sqrt{n}2^{-M} + 2\Delta_M + \sum_{L=K+1}^{M-1} \Delta_L \,.$$

Let M be the integer such that $2^{M-1} < n \le 2^M$. For this choice of M, we have

$$\Big\| \sup_{x\in[0,1]} |\mu_n(x) - \mu_n(\Pi_K(x))| \Big\|_1 \le C(\beta)n^{-1/2} + 2\sum_{L=K+1}^{M} \|\Delta_L\|_1 \,.$$

Hence, to prove (2.5), it suffices to prove that

$$\lim_{K\to\infty} \limsup_{n\to\infty} \sum_{L=K+1}^{M} \|\Delta_L\|_1 = 0 \,. \tag{2.10}$$

Choose $2 < p < 2(1+\delta)$. Clearly

$$\|\Delta_L\|_1^p \le \mathbb{E}(\Delta_L^p) \le \sum_{i=1}^{2^L} \mathbb{E}(|Z_n(](i-1)2^{-L}, i2^{-L}])|^p) \,. \tag{2.11}$$

We shall now control the term $\mathbb{E}(|Z_n(](i-1)2^{-L}, i2^{-L}])|^p)$ with the help of Proposition 3.1. Let $T_{i,k} = \mathbf{1}_{(i-1)2^{-L}<Y_k\le i2^{-L}}$ and $T_{i,k}^{(0)} = T_{i,k} - \mathbb{E}(T_{i,k})$. Clearly

$$\mathbb{E}(|Z_n(](i-1)2^{-L}, i2^{-L}])|^p) = \frac{1}{n^{p/2}}\mathbb{E}\Big(\Big|\sum_{k=1}^n T_{i,k}^{(0)}\Big|^p\Big) \le C\Big(a_i^{p/2} + n^{(2-p)/2}\Big(\|T_{i,0}^{(0)}\|_p^p + c_{i,1} + c_{i,2} + c_{i,3}\Big)\Big) \,,$$

where, according to Remark 3.1, for any $1 \le N \le n$,

$$
\begin{aligned}
a_i &= \frac{1}{2}\mathrm{Var}(T_{i,0}) + \sum_{k=1}^{N-1} |\mathrm{Cov}(T_{i,0}, T_{i,k})| + \sum_{k=N}^{n-1} \|T_{i,0}^{(0)} \mathbb{E}(T_{i,k}^{(0)} | \mathcal{M}_0)\|_{p/2} \,, \\
c_{i,1} &= \sum_{l=1}^{N-1} \sum_{k=0}^{l} \| |T_{i,0}^{(0)}|^{p-2} |T_{i,k}^{(0)}| \mathbb{E}(T_{i,k+l}^{(0)} | \mathcal{M}_k)\|_1 \,, \\
c_{i,2} &= \sum_{l=2}^{n} \sum_{k=(l-N)_+ +1}^{l-1} \| |T_{i,0}^{(0)}|^{p-2} \mathbb{E}(T_{i,l}^{(0)} T_{i,k}^{(0)} - \mathbb{E}(T_{i,l}^{(0)} T_{i,k}^{(0)}) | \mathcal{M}_0)\|_1 \,, \\
c_{i,3} &= \frac{1}{2} \sum_{k=1}^{n-1} \| |T_{i,0}^{(0)}|^{p-2} \mathbb{E}((T_{i,k}^{(0)})^2 - \mathbb{E}((T_{i,k}^{(0)})^2) | \mathcal{M}_0)\|_1 \,.
\end{aligned}
$$

Let us first bound up a_i. Taking the conditional expectation with respect to X_0, we have that

$$
|\mathrm{Cov}(T_{i,0}, T_{i,l})| \le 2\mathbb{E}(T_{i,0} b(X_0, l)) \,.
$$

With this upper bound, it follows that

$$
a_i \le \frac{1}{2}\mathbb{E}\Big((T_{i,0}\Big(1 + 4\sum_{l=1}^{\infty} b(X_0, l)\Big)\Big) + \sum_{l=N}^{\infty} \|T_{i,0}^{(0)} \mathbb{E}(T_{i,l}^{(0)} | \mathcal{M}_0)\|_{p/2} \,.
$$

Clearly

$$
\mathbb{E}\Big(T_{i,0}\Big(1 + 4\sum_{l=1}^{\infty} b(X_0, l)\Big)\Big) = C(\beta) \int \mathbf{1}_{(i-1)2^{-L} < F_Q(x) \le i2^{-L}} Q(dx) = C(\beta) 2^{-L} \,.
$$

In the same way, since $|T_{i,0}^{(0)}| \le 1$ and $b_1(\mathcal{M}_0, l) \le 1$,

$$
\sum_{l=N+1}^{\infty} \|T_{i,0}^{(0)} \mathbb{E}(T_{i,l}^{(0)} | \mathcal{M}_0)\|_{p/2} \le 2 \sum_{l=N+1}^{\infty} \|T_{i,0}^{(0)} b_1(\mathcal{M}_0, l)\|_{p/2} \le 2 \sum_{l=N+1}^{\infty} \big(\mathbb{E}(|T_{i,0}^{(0)}| b_1(\mathcal{M}_0, l))\big)^{2/p} \,.
$$

Hence, applying Hölder's inequality,

$$
\sum_{l=N+1}^{\infty} \|T_{i,0}^{(0)} \mathbb{E}(T_{i,l}^{(0)} | \mathcal{M}_0)\|_{p/2} \le 2\Big(\sum_{l=N}^{\infty} \beta_1(l)^{2/p} \Big)^{(p-2)/p} \Big(\sum_{l=N}^{\infty} \frac{\mathbb{E}(|T_{i,0}^{(0)}| b_1(\mathcal{M}_0, l))}{\beta_1(l)^{(p-2)/p}} \Big)^{2/p} \,.
$$

Since $\sum_{i=1}^{2^L} |T_{i,0}^{(0)}| \le 2$, we obtain that

$$
\sum_{i=1}^{2^L} \sum_{l=N}^{\infty} \frac{\mathbb{E}(|T_{i,0}^{(0)}| b_1(\mathcal{M}_0, l))}{\beta_1(l)^{(p-2)/p}} \le 2 \sum_{l=N}^{\infty} \beta_1(l)^{2/p} \,.
$$

Finally, we obtain the control

$$
\begin{aligned}
\sum_{i=1}^{2^L} a_i^{p/2} &\le C\Big(2^{-L(p-2)/2} + \Big(\sum_{l=N}^{\infty} \beta_1(l)^{2/p}\Big)^{p/2}\Big) \\
&\le C\big(2^{-L(p-2)/2} + N^{-(2(1+\delta)-p)/2}\big) \,. \qquad (2.12)
\end{aligned}
$$

On another hand, we have that

$$n^{(2-p)/2}\sum_{i=1}^{2^L}\|T_{i,0}^{(0)}\|_p^p \le n^{(2-p)/2}\sum_{i=1}^{2^L}\mathbb{E}(|T_{i,0}^{(0)}|) \le 2n^{(2-p)/2}\,. \tag{2.13}$$

For the term $c_{i,1}$, one gets

$$n^{(2-p)/2}\sum_{i=1}^{2^L} c_{i,1} \le 2n^{(2-p)/2}\sum_{l=1}^{N}\sum_{k=0}^{l}\sum_{i=1}^{2^L}\mathbb{E}(|T_{i,0}^{(0)}|b_1(\mathcal{M}_k,k+l)) \le 4n^{(2-p)/2}\sum_{l=1}^{N}(l+1)\beta_1(l)\,. \tag{2.14}$$

For the term $c_{i,2}$, one gets

$$n^{(2-p)/2}\sum_{i=1}^{2^L} c_{i,2} \le 2n^{(2-p)/2}\sum_{l=2}^{n}\sum_{k=(l-N)_+ +1}^{l-1}\sum_{i=1}^{2^L}\mathbb{E}(|T_{i,0}^{(0)}|b_2(\mathcal{M}_0,k)) \le 4n^{(2-p)/2}N\sum_{k=1}^{n}\beta_2(k)\,. \tag{2.15}$$

For the term $c_{i,3}$, note first that $(T_{i,k}^{(0)})^2 - \mathbb{E}((T_{i,k}^{(0)})^2) = (1-2\mathbb{E}(T_{i,k}))T_{i,k}^{(0)}$. Since $|1-2\mathbb{E}(T_{i,k})| \le 1$, it follows that

$$|\mathbb{E}((T_{i,k}^{(0)})^2 - \mathbb{E}((T_{i,k}^{(0)})^2)|\mathcal{M}_0)| \le |\mathbb{E}(T_{i,k}^{(0)}|\mathcal{M}_0)| \le 2b_1(\mathcal{M}_0,k)\,.$$

Hence, one gets

$$n^{(2-p)/2}\sum_{i=1}^{2^L} c_{i,3} \le 2n^{(2-p)/2}\sum_{k=1}^{n}\sum_{i=1}^{2^L}\mathbb{E}(|T_{i,0}^{(0)}|b_1(\mathcal{M}_0,k)) \le 4n^{(2-p)/2}\sum_{k=1}^{n}\beta_1(k)\,. \tag{2.16}$$

Now we take $N=[n^\epsilon]$, for $0<\epsilon<(p-2)/2$. From the bounds (2.12), (2.13), (2.14), (2.15) and (2.16), we infer that

$$\sum_{i=1}^{2^L}\mathbb{E}(|Z_n(](i-1)2^{-L},i2^{-L}])|^p) \le C\big(2^{-L(p-2)/2} + n^{-\epsilon(2(1+\delta)-p)/2} + n^{-(p-2)/2+\epsilon}\big)\,.$$

Consequently, since $M=O(\ln(n))$, it follows from (2.11) that

$$\limsup_{n\to\infty}\sum_{L=K+1}^{M}\|\Delta_L\|_1 \le C\sum_{L=K+1}^{\infty}2^{-L(p-2)/2p}\,,$$

and (2.10) easily follows. This completes the proof. □

Proof of Corollary 2.1. We have seen that, on the probability space $([0,1],\nu_\gamma)$, $(T_\gamma,\ldots,T_\gamma^n)$ is distributed as $(X_n,\ldots,X_1)$ where $(X_i)_{i\in\mathbb{Z}}$ is a stationary Markov chain with invariant measure ν_γ and transition kernel K_γ. Consequently, on the probability space $([0,1],\nu_\gamma)$, the empirical central limit theorem holds for $\sqrt{n}(F_{n,\gamma}-F_{\nu_\gamma})$ if and only if the it holds for $\sqrt{n}(F_n-F)$. It remains to check that the coefficients $\beta_2(k)$ of the Markov chain $(X_i)_{i\in\mathbb{Z}}$ satisfy the assumption of Theorem 2.1. From Theorem 3.1 in Dedecker and Prieur (2008), we see that

$$\beta_2(k) = O(k^{-a}) \quad \text{for any } a<(1-\gamma)/\gamma\,.$$

Since $\gamma<1/2$, one can choose a close enough to $(1-\gamma)/\gamma$, so that $\beta_2(k)=O(k^{-1-\delta})$ fore some $\delta>0$. Consequently Theorem 1.1 applies to the Markov chain $(X_i)_{i\in\mathbb{Z}}$. The covariance (2.1) of G_γ can be written

$$\mathrm{Cov}(G_\gamma(s), G_\gamma(t)) = \nu_\gamma(f_t^{(0)} \cdot f_s^{(0)}) + \sum_{k>0} \nu_\gamma(f_t^{(0)} K_\gamma^k f_s^{(0)}) + \sum_{k>0} \nu_\gamma(f_s^{(0)} K_\gamma^k f_t^{(0)}) \,,$$

which is exactly (2.2) thanks to the equality (1.1). Here the uniform continuity with respect to $d_{F_{\nu_\gamma}}$ is equivalent to the uniform continuity with respect to the usual distance on $[0,1]$, since ν_γ is equivalent to the Lebesgue measure. □

3. A Rosenthal Inequality for dependent random variables

In this section, we use the convention that $\sum_{i=j}^k a_i = 0$ if $j<k$.

Proposition 3.1. *Let $X_1, \dots, X_n$ be n real-valued random variables in $\mathbb{L}^p$ for some $p \in [2,3]$, with zero expectation. Let $S_n = X_1 + \dots + X_n$. For $1 \le i \le n$, let $\mathcal{F}_i = \sigma(X_1, \dots, X_i)$. For any $1 \le N \le n$, the following inequality holds*

$$\|S_n\|_p \le \Big(2(p-1)\sum_{i=1}^n \gamma_i\Big)^{1/2} + \Big(\sum_{i=1}^n \mathbb{E}(|X_i|^p) + p(p-1)\sum_{i=1}^n \big(\delta_{i,1} + \delta_{i,2} + \delta_{i,3}\big)\Big)^{1/p} \,,$$

where

$$\begin{aligned}
\gamma_i &= \frac{1}{2}\mathbb{E}(X_i^2) + \sum_{j=(i-N)_+ +1}^{i-1} |\mathbb{E}(X_i X_j)| + \sum_{j=1}^{i-N} \|X_j \mathbb{E}(X_i|\mathcal{F}_j)\|_{p/2} \,, \\
\delta_{i,1} &= \sum_{j=(i-N)_+ +1}^{i-1} \sum_{l=(2j-i)_+ +1}^{j} \||X_l|^{p-2}|X_j| \mathbb{E}(X_i|\mathcal{F}_j)\|_1 \,, \\
\delta_{i,2} &= \sum_{j=(i-N)_+ +1}^{i-1} \sum_{l=1}^{(2j-i)_+} \||X_l|^{p-2}\mathbb{E}(X_i X_j - \mathbb{E}(X_i X_j)|\mathcal{F}_l)\|_1 \,, \\
\delta_{i,3} &= \frac{1}{2}\sum_{j=1}^{i-1} \||X_j|^{p-2}\mathbb{E}(X_i^2 - \mathbb{E}(X_i^2)|\mathcal{F}_j)\|_1 \,.
\end{aligned}$$

Remark 3.1. *Assume that the X_i's of Proposition 3.1 are taken from a stationary sequence $(X_i)_{i\in\mathbb{Z}}$, and let $\mathcal{M}_i = \sigma(X_k, k \le i)$. One has $\gamma_i \le \tilde\gamma$, $\delta_{i,1} \le \delta_1$, $\delta_{i,2} \le \delta_2$ and $\delta_{i,3} \le \delta_3$, with*

$$\begin{aligned}
\tilde\gamma &= \frac{1}{2}\mathbb{E}(X_0^2) + \sum_{k=1}^{N-1} |\mathbb{E}(X_0 X_k)| + \sum_{k=N}^{n-1} \|X_0 \mathbb{E}(X_k|\mathcal{M}_0)\|_{p/2} \,, \\
\delta_1 &= \sum_{l=1}^{N-1} \sum_{k=0}^{l} \||X_0|^{p-2}|X_k| \mathbb{E}(X_{k+l}|\mathcal{M}_k)\|_1 \,, \\
\delta_2 &= \sum_{l=2}^{n} \sum_{k=(l-N)_+ +1}^{l-1} \||X_0|^{p-2}\mathbb{E}(X_l X_k - \mathbb{E}(X_l X_k)|\mathcal{M}_0)\|_1 \,, \\
\delta_3 &= \frac{1}{2}\sum_{k=1}^{n-1} \||X_0|^{p-2}\mathbb{E}(X_k^2 - \mathbb{E}(X_k^2)|\mathcal{M}_0)\|_1 \,.
\end{aligned}$$

Proof of Proposition 3.1. For $q > 0$, let $\psi_q(x) = |x|^q(\mathbf{1}_{x>0} - \mathbf{1}_{x\leq 0})$. Let $p \in [2,3]$. Applying Taylor's integral formula, and using that $||x|^{p-2} - |y|^{p-2}| \leq |x-y|^{p-2}$, one has that

$$\begin{aligned} |S_n|^p &= |S_{n-1}|^p + pX_n\psi_{p-1}(S_{n-1}) + p(p-1)\int_0^1 (1-t)X_n^2|S_{n-1}+tX_n|^{p-2}dt \\ &\leq |S_{n-1}|^p + pX_n\psi_{p-1}(S_{n-1}) + \frac{p(p-1)}{2}X_n^2|S_{n-1}|^{p-2} + p(p-1)\int_0^1 (1-t)t^{p-2}|X_n|^p dt\,. \end{aligned}$$

Consequently

$$\mathbb{E}(|S_n|^p) \leq \mathbb{E}(|S_{n-1}|^p) + p\mathbb{E}(X_n\psi_{p-1}(S_{n-1}) + \frac{p(p-1)}{2}\mathbb{E}(X_n^2|S_{n-1}|^{p-2}) + \mathbb{E}(|X_n|^p)\,. \tag{3.1}$$

Let us bound up all the terms on right hand.

The second order terms. Write first

$$X_n^2|S_{n-1}|^{p-2} = (X_n^2 - \mathbb{E}(X_n^2))|S_{n-1}|^{p-2} + \mathbb{E}(X_n^2)|S_{n-1}|^{p-2} = I_1 + I_2\,.$$

Now, with the convention $S_0 = 0$,

$$I_1 = \sum_{k=1}^{n-1}(X_n^2 - \mathbb{E}(X_n^2))(|S_k|^{p-2} - |S_{k-1}|^{p-2})\,.$$

Taking the conditional expectation with respect to $\mathcal{F}_k$ and using that $||x|^{p-2} - |y|^{p-2}| \leq |x-y|^{p-2}$, we obtain that

$$|\mathbb{E}(I_1)| \leq \sum_{k=1}^{n-1} \|\mathbb{E}(X_n^2 - \mathbb{E}(X_n^2)|\mathcal{F}_k)|X_k|^{p-2}\|_1\,. \tag{3.2}$$

The first order terms. For $1 \leq N \leq n$, write

$$X_n\psi_{p-1}(S_{n-1}) = X_n(\psi_{p-1}(S_{n-1}) - \psi_{p-1}(S_{n-N}) + X_n\psi_{p-1}(S_{n-N}) = I_3 + I_4\,.$$

Now

$$\begin{aligned} I_3 &= \sum_{k=n-N+1}^{n-1} X_n(\psi_{p-1}(S_k) - \psi_{p-1}(S_{k-1}) \\ &= (p-1)\sum_{k=n-N+1}^{n-1} X_nX_k\int_0^1 |S_{k-1}+tX_k|^{p-2}dt \\ &= (p-1)\Big(\sum_{k=n-N+1}^{n-1} X_nX_k|S_{k-1}|^{p-2} + \sum_{k=n-N+1}^{n-1} X_nX_k\int_0^1 (|S_{k-1}+tX_k|^{p-2} - |S_{k-1}|^{p-2})dt\Big) \\ &= J_1 + J_2\,. \end{aligned}$$

Taking the conditional expectation with respect to $\mathcal{F}_k$ and using that $||x|^{p-2} - |y|^{p-2}| \leq |x-y|^{p-2}$, we obtain that

$$|\mathbb{E}(J_2)| \leq \sum_{k=n-N+1}^{n-1} \||X_k|^{p-1}\mathbb{E}(X_n|\mathcal{F}_k)\|_1\,. \tag{3.3}$$

For the term J_1, write

$$\begin{aligned} J_1 &= (p-1)\Big(\sum_{k=n-N+1}^{n-1}\sum_{i=(2k-n)_++1}^{k-1} X_nX_k(|S_i|^{p-2}-|S_{i-1}|^{p-2}) + \sum_{k=n-N+1}^{n-1} X_nX_k|S_{(2k-n)_+}|^{p-2}\Big) \\ &= K_1 + J_3\,. \end{aligned}$$

Taking the conditional expectation with respect to $\mathcal{F}_k$ and using that $||x|^{p-2}-|y|^{p-2}| \le |x-y|^{p-2}$, we obtain that

$$|\mathbb{E}(K_1)| \le (p-1)\sum_{k=n-N+1}^{n-1}\sum_{i=(2k-n)_++1}^{k-1} \|X_k|X_i|^{p-2}\mathbb{E}(X_n|\mathcal{F}_k)\|_1 \tag{3.4}$$

For the term J_3, write

$$\begin{aligned} J_3 &= (p-1)\Big(\sum_{k=n-N+1}^{n-1}(X_nX_k-\mathbb{E}(X_nX_k))|S_{(2k-n)_+}|^{p-2} + \sum_{k=n-N+1}^{n-1}\mathbb{E}(X_nX_k)|S_{(2k-n)_+}|^{p-2}\Big) \\ &= K_2 + K_3\,. \end{aligned}$$

For the term K_2, write

$$K_2 = (p-1)\sum_{k=n-N+1}^{n-1}\sum_{i=1}^{(2k-n)_+}(X_nX_k-\mathbb{E}(X_nX_k))(|S_i|^{p-2}-|S_{i-1}|^{p-2})\,.$$

Taking the conditional expectation with respect to $\mathcal{F}_i$ and using that $||x|^{p-2}-|y|^{p-2}| \le |x-y|^{p-2}$, we obtain that

$$|\mathbb{E}(K_2)| \le (p-1)\sum_{k=n-N+1}^{n-1}\sum_{i=1}^{(2k-n)_+}\|\mathbb{E}(X_nX_k-\mathbb{E}(X_nX_k)|\mathcal{F}_i)|X_i|^{p-2}\|_1\,. \tag{3.5}$$

The remainder terms. It remains to control I_2, I_4 and K_3. For I_2 we have

$$\|I_2\|_1 \le \mathbb{E}(X_n^2)\|S_{n-1}\|_p^{p-2}\,. \tag{3.6}$$

For K_3, we have

$$\|K_3\|_1 \le (p-1)\sum_{k=n-N+1}^{n-1}|\mathbb{E}(X_nX_k)|\|S_{(2k-n)_+}\|_p^{p-2}\,. \tag{3.7}$$

For I_4, write

$$I_4 = \sum_{k=1}^{n-N} X_n(\psi_{p-1}(S_k)-\psi_{p-1}(S_{k-1})) = (p-1)\sum_{k=1}^{n-N}X_nX_k\int_0^1|S_{k-1}+tX_k|^{p-2}dt\,.$$

Taking the conditional expectation with respect to $\mathcal{F}_k$ and applying Hölder's inequality, we obtain that

$$|\mathbb{E}(I_4)| \le (p-1)\sum_{k=1}^{n-N}\|X_k\mathbb{E}(X_n|\mathcal{F}_k)\|_{p/2}\int_0^1\|S_{k-1}+tX_k\|_p^{p-2}dt\,. \tag{3.8}$$

End of the proof of Proposition 3.1. We proceed by induction on n. Clearly, Proposition 3.1 is true for $n = 1$. Assume that it holds for any positive integer k strictly less than n. Let

$$d_i = p(p-1)\Big(\frac{\mathbb{E}(|X_i|^p)}{p(p-1)} + \delta_{i,1} + \delta_{i,2} + \delta_{i,3}\Big)\,.$$

Applying the induction hypothesis on the right hand terms of (3.6), (3.7) and (3.8), we obtain that

$$\frac{1}{2}p(p-1)\|I_2\|_1 + p(|\mathbb{E}(I_4)| + \|K_3\|_1) \leq p(p-1)\gamma_n\Big(\Big(2(p-1)\sum_{i=1}^{n-1}\gamma_i\Big)^{1/2} + \Big(\sum_{i=1}^{n-1} d_i\Big)^{1/p}\Big)^{(p-2)}\,.$$

Now it is easy to infer that

$$\begin{aligned}\frac{1}{2}p(p-1)\|I_2\|_1 + p(|\mathbb{E}(I_4)| + \|K_3\|_1) \;\leq\;& \Big(\Big(2(p-1)\sum_{i=1}^{n}\gamma_i\Big)^{1/2} + \Big(\sum_{i=1}^{n-1} d_i\Big)^{1/p}\Big)^p \\ &-\Big(\Big(2(p-1)\sum_{i=1}^{n-1}\gamma_i\Big)^{1/2} + \Big(\sum_{i=1}^{n-1} d_i\Big)^{1/p}\Big)^p. \qquad (3.9)\end{aligned}$$

Applying the induction hypothesis on the first term on right hand in (3.1), and gathering (3.2), (3.3), (3.4), (3.5) and (3.9), we obtain that

$$\begin{aligned}\|S_n\|_p^p \;&\leq\; \Big(\Big(2(p-1)\sum_{i=1}^{n}\gamma_i\Big)^{1/2} + \Big(\sum_{i=1}^{n-1} d_i\Big)^{1/p}\Big)^p + d_n \\ &\leq\; \Big(\Big(2(p-1)\sum_{i=1}^{n}\gamma_i\Big)^{1/2} + \Big(\sum_{i=1}^{n} d_i\Big)^{1/p}\Big)^p.\end{aligned}$$

Hence, the result is true for any integer n, and the proof is complete. □

4. Appendix

Proposition 4.1. *Let $F_{\nu_{1/2}}(t) = \nu_{1/2}([0,t])$ and $F_{n,1/2}(t) = n^{-1}\sum_{i=1}^n \mathbf{1}_{T^i_{1/2}\leq t}$. On the probability space $([0,1], \nu_{1/2})$ the finite dimensional marginals of the process $(n/\ln(n))^{1/2}(F_{n,1/2} - F_{\nu_{1/2}})$ converges in distribution to those of the degenerated Gaussian process G defined by:*

$$\textit{for any } t \in [0,1], \quad G(t) = \sqrt{h(1/2)}(1 - F_{\nu_{1/2}}(t))\mathbf{1}_{t\neq 0}Z\,,$$

where Z is a standard normal and h is the density of $\nu_{1/2}$.

Proof of Proposition 4.1. It suffices to prove that for any piecewise constant function $f = \sum_{i=1}^k a_i\mathbf{1}_{[0,t_i]}$ with $t_i > 0$, $(n\ln(n))^{-1/2}S_n(f)$ converges in distribution to a Gaussian random variable with mean 0 and variance $h(1/2)(\sum_{i=1}^k a_i(1-F_{\nu_{1/2}}(t_i)))^2 = h(1/2)(f(0) - \nu_{1/2}(f))^2$.

Let g be some Lipschitz function which is equal to $\sum_{i=1}^k a_i$ on $[0,\epsilon]$ for $\epsilon > 0$, and such that $\nu_{1/2}(g) = \nu_{1/2}(f)$. Applying Gouëzel's result (2004) for Hölder functions (see the second point of the comments after his Theorem 1.3), we infer that $(n\ln(n))^{-1/2}S_n(g)$ converges in distribution to a Gaussian random variable with mean 0 and variance $h(1/2)(g(0)-\nu_{1/2}(g))^2 = h(1/2)(f(0)-\nu_{1/2}(f))^2$.

Now, the function $u = f - g$ is BV, equal to 0 on $[0, \epsilon]$, and such that $\nu_{1/2}(u) = 0$. For such a u, one can prove that

$$|\nu_{1/2}(u \cdot u \circ T_{1/2}^n)| = O(n^{-2}), \tag{4.1}$$

so that $n^{-1/2}\|S_n(u)\|_2$ converges, and $(n\ln(n))^{-1/2}\|S_n(u)\|_2$ converges to 0. Hence the two sequences $(n\ln(n))^{-1/2}S_n(f)$ and $(n\ln(n))^{-1/2}S_n(g)$ have the same limit distribution, and the result follows.

The proof of (4.1) is almost the same as that of Corollary 3.2 in Gouëzel (2007). It suffices to notice that the result of this corollary remains true if:

(1) Y is any interval of the form $[x_k, 1]$, where x_k is defined by: $x_0 = 1/2$ and $x_{k+1} = T_{1/2}^{-1}(x_k) \cap [0, 1/2]$.

(2) The Lebesgue measure is replaced by the invariant measure $\nu_{1/2}$. This means that the Perron Frobenius operator $\hat{T}_{1/2}$ with respect to the Lebesgue measure can be everywhere replaced by $K_{1/2}(f) = \hat{T}^{1/2}(fh)/h$, where h is the density of $\nu_{1/2}$ (recall that h is Lipschitz on $[x_k, 1]$).

Note that, with the same proof, one can see that if f is any BV function which is also Lipschitz in a neighbourhood of 0, $(n\ln(n))^{-1/2}S_n(f)$ converges in distribution to a Gaussian random variable with mean 0 and variance $h(1/2)(f(0) - \nu_{1/2}(f))^2$. □

Acknowledgments. Many thanks to Sébastien Gouëzel, who explained me how to prove Proposition 4.1 of the appendix. I also thank Herold Dehling for helpful discussions about the empirical distribution function of uniformly expanding maps.

References

[1] S. Borovkova, R. Burton and H. Dehling (2001), Limit theorems for functionals of mixing processes with applications to U-statistics and dimension estimation. *Trans. Amer. Math. Soc.* **353** 4261-4318.

[2] P. Collet, S. Martinez, and B. Schmitt (2004), Asymptotic distribution of tests for expanding maps of the interval. *Ergodic Theory Dynam. Systems* **24**, 707-722.

[3] J. Dedecker and C. Prieur (2007), An empirical central limit theorem for dependent sequences. *Stochastic Process. Appl.* **117**, 121-142.

[4] J. Dedecker and C. Prieur (2008), Some unbounded functions of intermittent maps for which the central limit theorem holds. Accepted for publication in *Alea*.

[5] H. Dehling, O. Durieu, and D. Volný (2007), New techniques for empirical process of dependent data. arXiv:0806.2941v1

[6] M. I. Gordin (1973), Abstracts of communication, T.1:A-K, International conference on probability theory, Vilnius.

[7] S. Gouëzel (2004), Central limit theorem and stable laws for intermittent maps. *Probab. Theory Relat. Fields* **128**, 82-122.

[8] S. Gouëzel (2007), A Borel-Cantelli lemma for intermittent interval maps. *Nonlinearity* **20** 1491-1497.

[9] H. Hennion and L. Hervé (2001), Limit theorems for Markov chains and stochastic properties of dynamical systems by quasi-compactness. *Lecture Notes in Mathematics* **1766**, Springer.

[10] C. Liverani, B. Saussol and S. Vaienti (1999), A probabilistic approach to intermittency. *Ergodic Theory Dynam. Systems* **19**, 671-685.

[11] E. Rio (2000), Théorie asymptotique des processus aléatoires faiblement dépendants. *Mathématiques et applications de la SMAI.* **31**, Springer.

[12] M. Rosenblatt (1956), A central limit theorem and a strong mixing condition. *Proc. Nat. Acad. Sci. U. S. A.* **42**, 43-47.

[13] A. W. van der Vaart and J. A. Wellner (1996), Weak convergence and empirical processes, Springer.

[14] L-S. Young (1999), Recurrence times and rates of mixing. *Israel J. Math.* **110**, 153-188.

J. Dedecker, Laboratoire de Statistique Théorique et Appliquée
Université paris 6, 175 rue du Chevaleret 75013 Paris, France.
email: jerome.dedecker@upmc.fr
home page: http://www.lsta.upmc.fr/dedecker.html